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Study of the Electric Quadrupole Transitions in $^{50-51}Mn$ Isotopes by Using *F742pn* and *F7cdpn* Interactions

دراسة الانتقالات رباعية القطب الكهربائي في نظائر $^{50-51}Mn$ باستعمال تفاعلات *F742pn* و *F7cdpn*

Ali Khalaf Hasan, Fatema Hameed Obeed and Azahr Nadham Rahim
Department of Physics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq

علي خلف حسن، فاطمة حميد عبيد، ازهار ناظم رحيم
قسم الفيزياء، كلية التربية للبنات، جامعة الكوفة، النجف، العراق

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ABSTRACT

The nuclear shell-model has been used to compute excitation levels of ground band and electric quadrupole transitions for $^{50-51}Mn$ isotopes in f-shell. In the present study, *f742pn* and *f7cdpn* effective interactions have been carried out in full f-shell by using Oxbash Code. The radial wave functions of the single-particle matrix elements have been calculated in terms of the harmonic oscillator (*Ho*) and Skyrme20 potentials. The predicted theoretical results have been compared with the available experimental data; it has been seen that the predicted results are in agreement with the experimental data. From the current results of the calculations, many predictions of angular momentum and parities of experimental states have been made, and the energy spectra predictions of the ground band and $B(E2;\downarrow)$ electric quadrupole transitions in $^{50-51}Mn$ isotopes of the experimental data are not known yet. In the nuclear shell-model calculations framework, energy levels have been determined for $^{50-51}Mn$ isotopes; new values of electric quadrupole transitions have been predicted in the studied results. This investigation increases the theoretical knowledge of all isotopes with respect to the energy levels and reduced transition probabilities.

الملخص

استعملنا أنموذج القشرة النووي لحساب مستويات الأثارة للحزمة الأرضية والانتقالات رباعية القطب الكهربائية لنظائر $^{50-51}Mn$ في القشرة –f. ، في الدراسة الحالية التفاعلات المؤثرة *f742pn* و *f7cdpn* نفذوا في القشرة الكاملة f- باستعمال كود الاوكسباش. تم حساب الدوال الموجية القطرية لعناصر المصفوفة للجسيم المنفرد في حدود جهود المذبذب التوافقي (*Ho*) وسكايرم 20. وتمت مقارنة النتائج النظرية المتوقعة مع البيانات التجريبية المتوافرة، وقد لوحظ أن النتائج المتوقعة تتفق بشكل جيد مع البيانات التجريبية. من النتائج الحالية للحسابات: هناك عدة تنبؤات للزخم الزاوي والتماثلات في الحالات التجريبية، فضلا عن توقعات أطياف الطاقة للحزمة الأرضية واحتمالات الانتقالات رباعية القطب الكهربائي $B(E2;\downarrow)$ في نظائر $^{50-51}Mn$ للبيانات التجريبية غير المعروفة حتى الآن. في نطاق حسابات أنموذج القشرة النووي، تم تحديد مستويات الطاقة لنظائر $^{50-51}Mn$ ، كما تم توقع قيم جديدة للانتقالات رباعية القطب الكهربائي في النتائج المدروسة. هذا الاقتراح يزيد المعرفة النظرية لجميع النظائر فيما يتعلق بمستويات الطاقة و احتمالات الانتقال المختزلة.

KEYWORDS
الكلمات المفتاحية
Effective charges, excitation levels, Oxbash Code, shell-model, ground band, angular momentum, and parity
الشحنات المؤثرة، مستويات التهيج , كود الاوكسباش، أنموذج القشرة، الحزمة الأرضية ،الزخم الزاوي والتماثل

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1. Introduction

The nuclear structure of neutron-rich nuclei above ^{40}Ca magic nucleus has been the recent experimental focus. The mechanism can cause changes in nuclear structure as neutron numbers increase in any nuclear system mechanism (it is an open question)!!(Srivastava and Mehrotra 2010). Neutron-rich f-shell isotopes are of private interest, too. In astrophysics, like the electron capture rate in supernovae explosion, experimental and theoretical explanations can be achieved as a satisfactory description for these isotopes (Kaneko et al., 2008). Previously, several neutron-rich nuclei in $f_{7/2}$-shell studies have been carried out and investigated by many researchers, including the three-body force manifestations in spectra, binding energies, seniority mixing, particle-hole symmetry, electromagnetic and particle transition rate (Volya, 2009). The energy states and electric quadrupole transitions for the $0f_{7/2}$ proton-shell nuclei (^{50}Ti, ^{51}V, ^{52}Cr, ^{53}Mn, ^{54}Fe, and ^{55}Co) have been calculated within the spherical shell-model framework, assuming a ^{48}Ca as a closed core (Saaymant and Irvine, 1976), as well as $f_{7/2}$ shell-model study ($^{43-46}$Ca and $^{43-45}$Sc) in pure $f^n_{7/2}$ configuration (Muto et al.,1978). Hundreds of years ago, many models and theories were built to describe and illustrate the experimental observations of the nuclear structure and reaction properties for various nuclei (Bacca, 2016). One of those nuclear models is the nuclear shell-model, which has successfully explained the nuclear structure for several nuclei in different areas of the periodic table, such as Ca, Ti, Cr, and Fe (Gambhir et al .,1982). The nuclear shell-model is considered the primary theoretical tool for understanding the nuclei properties. It can be used for preparing qualitative understanding due to its simple single-particle form, which is essentially based on the effective interaction use, actually because of its well-known form. Within the shell-model approach, just the particles outside the close core can make up the filled shells (valence nucleons), which are considered active nucleons, and calculations are achieved in truncated Hilbert space called the model space (Gargano et al., 2014). Nuclear shell-model calculations are carried out within a model space in which the valence nucleons are limited to distribute over a few orbits that are incapable of reproducing the measured static moments or transition strengths and other nuclear properties (Radhi and Bouchebak, 2003). Multipole transition probability, binding energies, separation energies, beta decay, quadrupole moments, and magnetic moments have been calculated based on nuclear shell-model codes such as OXBASH (Brown et al., 2005), NSHELL(Brown and Rae, 2007), ANTOINE (Caurier and Nowacki, 1999), NATHAN (Caurier et al., 1999) and NuShellX@MSU (Brown and Rae, 2004). These had been vastly utilized besides the single-particle wave function, the two-body matrix element (TBME) of variety shell-model spaces, and different effective interactions had been suggested depending on two-body interaction strength. TBME has been

Corresponding Author: Fatema Hameed Obeed fatimahh.alfatlawi@uokufa.edu.iq, 0096407817322815 المؤلف المراسل: فاطمة حميد عبيد

represented as an important action in calculating the nuclear properties (Brown and Rae, 2004). However, in the current calculations, the Oxbash Code has been performed to find some nuclear properties for nuclear structure. This is a set of codes for carrying out shell-model calculations with dimensions up to 50,000 in the *J-T* scheme and about 2,000,000 in the M-scheme. Oxbash comes with a library of model spaces and interactions (Hasan and Obeed, 2017). The current study aims to compute the energy levels and electric quadrupole transitions for $^{50-51}Mn$ isotopes by using effective interactions f742pn and f7cdpn in f-shell then make a comparison of the current results with the available experimental values.

2. Theory

In this work, nucleons of $^{50-51}$Mn isotopes have been distributed to the nucleons in $f_{7/2}$-shell, and these isotopes have proton and neutron numbers lying above the magic numbers Z=20 and N=20. Although the number of active valence nucleons is low enough in these nuclei to allow for a full shell-model description, these nuclei forms can be an interesting island in the nuclide chart. An essential objective of nuclear structure theory has carried out shell-model studies with single-particle (SP) energies and residual two-body interactions that are both derived from a realistic nucleon-nucleon interaction (Coraggio et al., 2007). Generally, a quantum mechanics solution to the Schrödinger equation have been necessary steps to a specified appropriate Hamiltonian, so that a standard shell-model of effective Hamiltonian can be written as:

$$H = \sum_a \varepsilon_a n_a + \sum_{a\le b,c\le d} \sum_{JT} V_{JT}(ab;cd) \sum_{MT_Z} A^{+}_{JMTT_Z}(ab) A_{JMTT_Z}(cd)] \quad (1)$$

Where n_a: describes the number operator of an orbit (a) of quantum numbers (n_a, l_a, j_a) but the symbol ($A_{JMTT_Z}(cd)$) denotes the creation operator of nucleon-pairs in orbits (a and b) with spin quantum numbers (JM) and isospin quantum numbers (T, T_Z), but the last terms in equ. (1) can be written as the following (Honma et al., 2002:135c):

$$\sum_{k<l} V_{kl} = [\sum_{a\le b,c\le d} \sum_{JT} V_{JT}(ab;cd) \sum_{MT_Z} A^{+}_{JMTT_Z}(ab) A_{JMTT_Z}(cd)] \quad (2)$$

Where $\sum_{k<l} V_{k\ell}$ is expressed on residual two-body interactions and can be re-written as:

$$\sum_{k<l} V_{k\ell} = \langle j_1 j_2 | V_{12} | j_3 j_4 \rangle \quad (3)$$

From the equ. (1, 2, and 3) it can get on the following :

$$H = \sum_a \varepsilon_a n_a + \langle j_1 j_2 | V_{12} | j_3 j_4 \rangle = \sum_{k=\ell}^{A} H_0 + \langle j_1 j_2 | V_{12} | j_3 j_4 \rangle \quad (4)$$

By separating the summations into core and valence contributions in eq. (4), the eq. (4) can be re-written:

$$H = H_{core} + H_1 + H_2 + \langle j_1 j_2 | V_{12} | j_3 j_4 \rangle \quad (5)$$

Here, H_{core} implies on all of the interactions of nucleons making up the close core, while H_1 and H_2 refer to the single-particle contributions from particles (1 and 2) and $\langle j_1 j_2 | V_{12} | j_3 j_4 \rangle$ are the residual interactions describing all interactions between particles (1 and 2) as well as any interaction with core nucleons. Inserting this form of the Hamiltonian into the Schrödinger equation yields an analogous expression for the energy (Lawson, 1980:43):

$$E = E_{core} + E_1 + E_2 + \langle j_1 j_2 | V_{12} | j_3 j_4 \rangle \quad (6)$$

The terms of (equ. 6) can be interpreted as follows. E_{core} is the binding energy of the core nucleus, E_1 and E_2 are defined as the single-particle energies of orbitals outside the core, and $\langle j_1 j_2 | V_{12} | j_3 j_4 \rangle$ is the residual interaction. Equation (6) is necessary to calculate the energy levels for pure configurations only. The excitation energy levels, wave functions, effective charges, and core polarization effects are necessary inputs to calculate the reduced electric quadrupole transition probabilities, $B(E2\downarrow)$. Reduced electric quadrupole transition probabilities are considered the observable necessity, and quadrupole moment ratios within the low-lying state bands can provide more information about the nuclear structure (Hasan, 2018:191). The present theoretical results of the reduced quadrupole transition probability $B(E2\downarrow)$ can be carried out from the recent calculations study of the reduced electric matrix element and from the initial and final nuclear states that are given in the following equation (Ali, 2018:1):

$$T(EJ) = \langle J_f \| \sum_k e(k) \hat{Q}_J(\vec{r})_k \| J_i \rangle_{MS} \quad (7)$$

e(k): represents the electric charge for the k-th nucleon and e(k) = 0 for the neutron.

While the electric matrix element can be represented in terms of the model space matrix elements only by assigning effective charges (e_{eff} (t_z)) to the neutrons and protons (Pandya and Singh, 1974:66)

$$T(EJ) = \sum_{t_z} e_{eff}(t_z) \langle J_f || \hat{Q}_2(\vec{r}, t_z z) || J_i \rangle_{MS} \quad (8)$$

It can be the expression for the effective charges to include the neutron excess explicitly (Brown, 2001:575):

$$e_{eff}(t) = e(t) + e\delta e(t), \delta e(t) = \frac{Z}{A} - \frac{0.32(N-Z)}{A} - 2t_z [0.32 - \frac{0.3(N-Z)}{A}] \quad (9)$$

Where $\delta e(t_Z)$ is expressed from the polarization charge for nucleon.

The reduced quadrupole transition probability can be according to the relation:

$$B(E2; I_i \to I_f \downarrow) = \frac{\langle I_f \| T^{E2} \| I_i \rangle^2}{2I_i + 1} \quad (10)$$

The chi-square between the computed and the experimental values for each energy level can be calculated according to the following equation (Walker, 1995:62):

$$\chi^2 = \sum \frac{(E_{exp.} - E_{theor.})^2}{E_{theor.}} \quad (11)$$

But chi-square conditions are not applied for reduced electric quadrupole transition probability.

3. Results and Discussion

In this paper, some properties of nuclear structure for ground bands of $^{50-51}Mn$ isotopes have been calculated using Oxbash Code. Valence nucleons of these isotopes are distributed in f-shell. In these calculations, *f7pn* model space, *f742pn,* and *f7cdpn* interactions have been utilized to estimate the energy levels and the reduced quadrupole transition probability for the isotopes mentioned above. The reduced quadrupole transition probabilities have been carried out using harmonic oscillator (*Ho*) and Skyrme20 potentials at the isotopes' Oxbash Code under this study. So the $^{50,51}Mn$ isotopes can be discussed as follows.

3.1. Energy Levels:

Table.1 illustrates the comparison between the predicted theoretical results of positive parity states in ^{50}Mn nucleus for *f742pn* and *f7cdpn* interactions and the recent experimental data. This nucleus consists of (25 proton) and (25 neutron) outside ^{40}Ca magic core, which have filled out the ($(1f_{7/2})$shell). In table 1, theoretical values of $\{1_1^+$ and $2_1^+\}$ states of f742pn and f7cdpn interactions have agreed very well with experimental energies {0.651 and 0.800} *MeV* respectively (Chen and Singh, 2019: n/a). The experimental energies

{0.659 and 1.765MeV} appeared in good agreement with expected theoretical values at both *f742pn* and *f7cdpn* interactions in order to confirm the states {3_1^+ and 4_1^+} of the above experimental energies. The theoretical states {7_1^+, 9_1^+, and 11_1^+} have confirmed the experimental values {1.030, 2.534, and 4.585} and *MeV* respectively . In current studied results, the states {8_1^+} are determined at the experimental energies {3.177} *MeV*, which are unknown at spins and parities experimentally. New values at energies and states in both *f742pn* and *f7cdpn* interactions have been predicted in the studied present results as the states {6_1^+, 10_1, and 12_1^+} have not been known at the energy and state values experimentally.

Table 1: Comparison between theoretical and experimental excitation energies (*MeV*) (Chen and Singh, 2019:n/a) for ^{50}Mn nucleus by using *F742pn* and *F7cdpn* Interactions

$J^+_{theor.}$	$E_{(x)theor.}$ *F742pn.* Interaction	$E_{(x)theor.}$ *F7cdpn.* Interaction	$J^+_{Exp.}$	$E_{(x)}$ Exp.	χ^2 *F742pn.* Interaction	χ^2 *F7cdpn.* Interaction	Status
0_1^+	0.000	0.000	0^+	0.000	0.0848	0.1120	acceptance
5_1^+	0.355	0.464	5^+	0.225			
1_1^+	0.523	0.637	1^+	0.651			
3_1^+	0.611	0.718	(6^+)	0.659	0.0309	0.0492	determination
6_1^+	0.903	1.001	------	------			
7_1^+	1.139	1.203	(7^+)	1.030			
2_1^+	1.148	1.190	2^+	0.800	0.0025	0.0007	confirmation
4_1^+	1.878	1.979	$(3^+,7^+)$	1.765			
9_1^+	2.809	2.863	(9^+)	2.534			
8_1^+	3.026	3.099	------	3.177	0.1182	0.1619	total
11_1^+	4.658	4.682	(11^+)	4.585			
10_1^+	5.348	5.375	------	------			
12_1^+	7.958	7.936	------	------			

While ^{51}Mn isotope has 25 protons and 26 neutrons, which have occupied the $1f_{7/2}$ shell above ^{40}Ca magic core. The compression between the theoretical and experimental results (Jimin. and Xiaolong, 2017:n/a) are shown in table.2. From these comparisons, there has been a good agreement for theoretical energy values at states {$11/2_1^-$, $15/2_1^-$, $17/2_1^-$, $19/2_1^-$, and $23/2_1^-$} with experimental data. In this study, calculations of the state {$1/2_1^-$} have been affirmed for experimental energy {3.423} *MeV* and have not been affirmed at states experimentally. The theoretical states like {$25/2_1^-$} have been determined as new states of available experimental data {8.498 MeV}. In this nucleus, new levels of energies and states have appeared in this study at both *f742pn* and *f7cdpn* states interactions like states {$3/2_1^-$, $13/2_1^-$, and $21/2_1^-$} and disappeared in the experimental levels scheme.

Table 2.Comparison between theoretical and experimental excitation energies (*MeV*) (Jimin. and Xiaolong, 2017:n/a) for ^{51}Mn nucleus by using F742pn and F7cdpn Interactions

$J^-_{theor.}$	$E_{x,theor.}$ *F742pn.* Interaction	$E_{x,theor.}$ *F7cdpn.* Interaction	$J^-_{Exp.}$	$E_{x,Exp.}$	χ^2 *F742pn.* Interaction	χ^2 *F7cdpn.* Interaction	Status
$5/2_1^-$	0.000	0.000	$5/2^-$	0.000	0.0136	0.0072	acceptance
$7/2_1^-$	0.093	0.054	$7/2^-$	0.237			
$3/2_1^-$	1.526	1.506	------	------			
$9/2_1^-$	1.529	1.508	$9/2^-$	1.139	0.0022	0.0022	determination
$11/2_1^-$	1.571	1.523	$11/2^-$	1.488			
$15/2_1^-$	3.236	3.177	$15/2^-$	3.250			
$1/2_1^-$	3.290	3.291	$(1/2^-,7/2)$	3.423	0.0003	0.0047	confirmation
$13/2_1^-$	3.490	3.455	------	------			
$17/2_1^-$	3.720	3.718	$17/2^-$	3.680			
$19/2_1^-$	4.222	4.170	$19/2^-$	4.139	0.0161	0.0141	total
$21/2_1^-$	6.338	6.281	------	------			
$23/2_1^-$	6.522	6.445	$23/2^-$	6.471			
$25/2_1^-$	8.453	8.305	------	8.498			

3.2. Reduced Electric Quadrupole Transition Probability:

The radial wave functions for the single-particle matrix elements have been calculated using the *(Ho*), default *SK20* potentials, and the effective nucleon charges. Oxbash Code has been used to calculate the reduced electric quadrupole transition for estimating a better fitting with the experimental values. The (*Ho*) and *SK20* parameters are size parameter values of ($^{50-51}Mn$ isotopes) and are *b= 1.928 fm of ^{50}Mn nucleus and b= 1.9235 fm of ^{51}Mn nucleus* and Skyrme parameters *are* t_0, t_2, t_3, w_0, *a*, x_0, x_1, x_2, x_3 of values {1445.322,246.867,-131.786, 12103.863,148.637, 0.5,0.340, 0.580, 0.127, 0.030} respectively. There has been excellent acceptance between the theoretical values of the transition strength *B(E2↓)* of $^{50-51}Mn$ isotopes at both *f742pn* and *f7cdpn* interactions and the experimental data. Tables 3–6 display the comparison between theoretical values of the electric quadrupole transition probability and the experimental data. For ^{50}Mn nucleus, tables 3 and 4 show that the experimental values of B(E2↓) from{$2_1^+ \rightarrow 0_1^+$ } and {$(9_1^+)\longrightarrow(7_1^+)$} have agreed with the predicted theoretical values of *f742pn* and *f7cdpn* interactions. While tables 5 and 6 show the comparison between the predicted theoretical values of the electric quadrupole transition probability and experimental data of ^{51}Mn nucleus. From this comparison, an excellent agreement of theoretical values of the experimental data has appeared, such as {$7\backslash2_1^-\longrightarrow5\backslash2_1^-$,$9\backslash2_1^-\longrightarrow7\backslash2_1^-$,$11\backslash2_1^-\longrightarrow7\backslash2_1^-$,$3\backslash2_1^-\longrightarrow7\backslash2_1^-$ and $1\backslash2_1^-\longrightarrow5\backslash2_1^-$}. New values of the quadrupole electric transitional probability of $^{50-51}Mn$ nuclei have been expected in this studied results like *B(E2↓)* from {$6_1^+\longrightarrow4_1^+$,$(8_1^+)\longrightarrow(6_1^+)$,$10_1^+\longrightarrow 8_1^+$and $12_1^+\longrightarrow10_1^+$} of ^{50}Mn nucleus but of ^{51}Mn nucleus the values have been {$15\backslash2_2^-\longrightarrow11\backslash2_1^-$,$19\backslash2_2^-\longrightarrow15\backslash2_1^-$,$21\backslash2_1^-\longrightarrow17\backslash2_1^-$,$23\backslash2_2^-\longrightarrow19\backslash2_1^-$ and $25\backslash2_1^-\longrightarrow21\backslash2_1^-$}; these values have not been known in the experimental data.

Table 3.Comparison between theoretical and experimental values(Chen and Singh, 2019:n/a) of electric quadrupole transition for ^{50}Mn nucleus by using *F742pn* interaction

$J_i\rightarrow J_f$	$B(E2\downarrow)_{theor.}$ $B(E2\downarrow)$ e^2fm^4 *Ho* e_p=1.523e e_n=0.536e	$B(E2\downarrow)_{theor.}$ $B(E2\downarrow)$ W.u *Ho* e_p=1.523e e_n=0.536e	$B(E2\downarrow)_{theor.}$ $B(E2\downarrow)$ e^2fm^4 *SK20* e_p=1.523e e_n=0.536e	$B(E2\downarrow)_{theor.}$ $B(E2\downarrow)$ W.u *SK20* e_p=1.523e e_n=0.536e	$B(E2\downarrow)_{Exp.}$ W.u
$2_1^+\rightarrow0_1^+$	153.1	13.992	150.9	13.791	13.9^{+24}_{-22}
$4_1^+\rightarrow2_1^+$	0.3272	0.0299	0.3316	0.0303	>15
$6_1^+\rightarrow4_1^+$	37.20	3.399	36.65	3.349	------
$8_1^+\rightarrow6_1^+$	79.64	7.278	78.48	7.172	------
$10_1^+\rightarrow8_1^+$	0.003658	0.000334	0.003604	0.000329	------
$12_1^+\rightarrow10_1^+$	11.76	1.074	11.58	1.058	------
$3_1+\rightarrow1_1^+$	149.9	13.700	147.7	13.498	64^{+30}_{-21}
$9_1^+\rightarrow7_1^+$	163.5	14.943	161.1	14.723	12.9^{+24}_{-17}

Table 4: Comparison between theoretical and experimental values(Chen and Singh, 2019:n/a) of electric quadrupole transition for ^{50}Mn nucleus by using F7cdpn interaction

$J_i\rightarrow J_f$	$B(E2\downarrow)_{theor.}$ *B(E2↓)* e^2fm^4 *Ho* e_p=1.523e e_n=0.536e	$B(E2\downarrow)_{theor.}$ $B(E2\downarrow)$ W.u *Ho* e_p=1.523e e_n=0.536e	$B(E2\downarrow)_{theor.}$ *B(E2)* e^2fm^4 *Sk20* e_p=1.523e e_n=0.536e	$B(E2\downarrow)_{theor.}$ *B(E2)* W.u *Sk20* e_p=1.523e e_n=0.536e	$B(E2\downarrow)_{Exp.}$ W.u
$2_1^+\rightarrow0_1^+$	153.1	13.992	150.9	13.791	13.9^{+24}_{-22}
$4_1^+\rightarrow2_1^+$	1.039	0.0949	1.040	0.0951	>15
$6_1^+\rightarrow4_1^+$	36.99	3.381	36.45	3.331	------
$8_1^+\rightarrow6_1^+$	78.73	7.1954	78.25	7.1516	------
$10_1^+\rightarrow8_1^+$	0.2357	0.0215	0.2309	0.0211	------
$12_1^+\rightarrow10_1^+$	0.2002	0.0182	0.2042	0.0186	------
$3_1^+\rightarrow1_1^+$	148.5	13.572	146.3	13.371	64^{+30}_{-21}
$9_1^+\rightarrow7_1^+$	163.4	14.933	161.0	14.714	12.9^{+24}_{-17}

Table 5: Comparison between theoretical and experimental values (Jimin. and Xiaolong, 2017:n/a) of electric quadrupole transition for ^{51}Mn nucleus by using *F742pn* interaction

$J_i\rightarrow J_f$	$B(E2\downarrow)_{theor.}$ *B(E2↓)* e^2fm^4 *Ho* e_p=2.168e e_n=1.275e	$B(E2\downarrow)_{theor.}$ $B(E2\downarrow)$ W.u *Ho* e_p=2.168e e_n=1.275e	$B(E2\downarrow)_{theor.}$ B(E2) e^2fm^4 *SK20* e_p=2.191e e_n=1.311e	$B(E2\downarrow)_{theor.}$ *B(E2)* W.u *SK20* e_p=2.191e e_n=1.311e	$B(E2\downarrow)_{Exp.}$ W.u
$7\backslash2_1^-\rightarrow5\backslash2_1^-$	528.6	46.973	527.6	46.884	47± 13
$9\backslash2_1^-\rightarrow7\backslash2_1^-$	221.5	19.683	220.0	19.549	27 ± 10
$9\backslash2_1^-\rightarrow5\backslash2_1^-$	122.8	10.912	122.0	10.841	156
$11\backslash2_1^-\rightarrow7\backslash2_1^-$	315.5	28.036	313.6	27.867	21± 6
$3\backslash2_1^-\rightarrow7\backslash2_1^-$	110.0	9.774	109.2	9.703	12 ± 10
$1\backslash2_1^-\rightarrow5\backslash2_1^-$	42.78	3.801	43.58	3.872	4.6 ± 15
$17\backslash2_1^-\rightarrow13\backslash2_1^-$	16.75	1.488	16.66	1.480	0.092 ± 4
$15\backslash2_1^-\rightarrow11\backslash2_1^-$	342.3	30.417	340.1	30.222	------

$19\backslash2_1^- \rightarrow 15\backslash2_1^-$	136.6	12.138	135.8	12.067	------
$21\backslash2_1^- \rightarrow 17\backslash2_1^-$	103.2	9.170	102.8	9.135	------
$23\backslash2_1^- \rightarrow 19\backslash2_1^-$	228.1	20.269	227.0	20.171	------
$25\backslash2_1^- \rightarrow 21\backslash2_1^-$	139.9	12.431	139.3	12.378	------

Table 6: Comparison between theoretical and experimental values (Jimin. and Xiaolong, 2017:n/a) of electric quadrupole transition for ^{51}Mn nucleus by using *F7cdpn* interaction

$J_i \rightarrow J_f$	$B(E2\downarrow)_{theor.}$				$B(E2\downarrow)_{Exp.}$ W.u
	$B(E2\downarrow)$ $e^2 fm^4$	$B(E2\downarrow)$ W.u	$B(E2)$ $e^2 fm^4$	$B(E2)$ W.u	
	Ho e_p=2.193e e_n=1.315	**Ho** e_p=2.193e e_n=1.315e	**Sk20** e_p=2.193e e_n=1.315e	**Sk20** e_p=2.193e e_n=1.315e	
$7\backslash2_1^- \rightarrow 5\backslash2_1^-$	526.7	46.804	523.6	46.528	47 ± 13
$9\backslash2_1^- \rightarrow 7\backslash2_1^-$	219.5	19.505	218.1	19.381	27± 10
$9\backslash2_1^- \rightarrow 5\backslash2_1^-$	122.6	10.894	121.8	10.823	156
$11\backslash2_2^- \rightarrow 7\backslash2_1^-$	317.4	28.205	315.5	28.036	21± 6
$3\backslash2_1^- \rightarrow 7\backslash2_1^-$	229.2	20.367	227.8	20.243	<0.14
$3\backslash2_1^- \rightarrow 7\backslash2_2^-$	108.3	9.623	107.5	9.552	12 ± 10
$1\backslash2_2^- \rightarrow 5\backslash2_1^-$	42.69	3.793	43.44	3.860	4.6± 15
$3\backslash2_2^- \rightarrow 7\backslash2_1^-$	0.1509	0.0134	0.1566	0.0139	2.6± 15
$17\backslash2_1^- \rightarrow 13\backslash2_1^-$	19.99	1.776	19.87	1.765	0.092± 4
$15\backslash2_2^- \rightarrow 11\backslash2_1^-$	343.8	30.551	341.6	30.355	------
$19\backslash2_2^- \rightarrow 15\backslash2_1^-$	134.1	11.916	133.2	11.836	------
$21\backslash2_1^- \rightarrow 17\backslash2_1^-$	106.1	9.428	105.6	9.383	------
$23\backslash2_2^- \rightarrow 19\backslash2_1^-$	227.9	20.251	226.7	20.145	------
$25\backslash2_1^- \rightarrow 21\backslash2_1^-$	141.7	12.591	133.7	11.881	------

4. Conclusions

The calculations for the positive and negative parity levels of $^{50\text{-}51}Mn$ isotopes have indicated that the nuclear shell-model and *f742pn* and *f7cdpn* interactions have been very successful in explaining the nuclear structure of these nuclei due to the less complex interactions of valence nucleons. There has been an acceptance agreement of the studied theoretical results and experimental data of all properties that have been calculated in the present results. Also, some spins and parities of these isotopes have been confirmed and determined. This showed in chi-square values that appeared very small when the observed data lies close to the expected data from through agreement values of them as well as determination, confirmation, and total. Furthermore, new values of energy levels and the quadrupole electric transitional probability, which have been predicted in this study's results, have not been known in the experimental data. These values can add more information for theoretical knowledge for all isotopes in this work. All calculations have been compared with the available experimental data; furthermore, they are very functional for compiling nuclear data tables. It has been concluded that the present calculations have consolidated two forms of effective interactions and two types of residual interactions for (f-shell) to calculate the energy levels and the electric quadrupole transition probability whose set has been appropriated for providing theoretical predictions of the nuclear structure for each isotope and can achieve a high degree of conformity with the available results, also succeeded in describing the available transition data.

Biographies

Ali Khalaf Hasan

Department of Physics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq, 0096407802461719, alikh.alsinayyid@uokufa.edu.iq

Dr. Hasan is an Iraqi professor; he obtained his Ph.D. in nuclear physics from the University of Basra-Iraq in 2009. His main research interests are quantum, theoretical, nuclear, and radiation physics. He participated in many local scientific conferences in Iraq. He published around 50 papers in scientific journals in Iraq and international journals within Scopus containers, such as the *International Journal of Physical Sciences, Ukrainian Journal of Physics International Journal of Current Research*, and *AIP Conference Proceedings*. ORCID: 0000-0002-8126-5179

Fatema Hameed Obeed

Department of Physics, Faculty of Education for Girls, University of Kufa, Najaf, Iraq, fatimahh.alfatlawi@uokufa.edu.iq, 0096407817322815

Mrs. Obeed is an Iraqi assistant professor. She obtained a master's in nuclear physics from Kufa University-Iraq in 2010. Her main research interests are theoretical study in nuclear physics of nuclear structure using Fortran and MATLAB programming codes. She obtained training in teaching methods and computer education. Obeed participated in many local and international scientific conferences. She published six research papers in local scientific journals in Iraq and international journals within Scopus containers. ORCID: 0000-0003-2076-0376.

Azhar Nadham Rahim

Physics Department, College of Education for Pure Sciences, University of Karbala, Karbala, Iraq, azhar.n@uokerbala.edu.iq, 0097407728781526

Mrs. Rahim is an Iraqi assistant teacher. She obtained a master's in nuclear physics from the University of Kufa-Iraq in 2018. Her research interests are nuclear structure study of nuclei using Fortran programming codes in addition to theoretical and radiation physics. She obtained training for teaching methods and computer education. Rahim has published research papers in international journals within Scopus containers. The central theme of this research was to calculate and describe the properties of the nuclei. ORCID: 0000-0001-9326-669X

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